# Theory of optical spontaneous emission rates in layered structures

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This paper presents expressions for the optical power and fields radiated by an oscillating dipole in layered structures, with loss or gain, that are easy to evaluate. For structures without loss or gain, it is discussed that the radiated power is a direct measure for the local density of modes, whereas for structures with loss or gain, this correspondence no longer holds due to interaction between the dipole and evanescent modes. The presented theory is illustrated with a number of examples.

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## I. INTRODUCTION

It is well known [1,2] that the transition rate of spontaneously emitting atoms or molecules, as well as the corresponding radiated optical field distribution, depend on the local density of optical states corresponding to the electronic energy of the excited state. The latter corresponds, from the viewpoint of waveguide theory, to the local density of optical modes (LDOM). Theoretical and experimental work in the above field focuses among others on emitters in photonic crystals [3], grated structures [4], two-dimensional (2D) cylindrically symmetric structures [5,6] structures optimized for collecting surface generated fluorescence [7], and layered structures [8,9]. Scientific studies on the latter have been performed because also in such structures the environmental effects on the spontaneous emission (SE) rates are considerable [8–10], and because the relative simplicity of such basic structures allows for insight into the physical phenomenon of environmental dependence of SE.

Theory of SE rates in layered structures has been published by several authors for structures without [8,10–12] and with loss [13–16]. Approximate treatments, including the effect of guided modes, have been presented by [10] using Green's functions and by [11] on the basis of a Hertz vector formalism. The treatment presented by [11], as well as the one by [12], require an evaluation of guided modes occurring in the (real) structure. Theory of power emitted by sources in lossy layered structures has been presented by [13–15], on the basis of Fourier analysis. In [16], a detailed derivation is given of Green's functions for layered structures, which may show gain. In the present paper, it is discussed how to derive full expressions for both power, radiated by a classical oscillating dipole, and the corresponding field distribution for layered structures, which may also show absorption and optical gain in part of the layers. The evaluation of radiated fields and power is performed by simple contour integration in the complex $k_{\|}$ plane, $k_{\|}$ being the in-plane modal wave number, along a contour that is well-positioned relative to the poles representing guided mode solutions. The relation between the radiated power and the LDOM is discussed. As indicated in the text, part of the expressions has been derived before by [8,11], considering radiation modes, and by [16] for lossy structures; the derivations of these are also given here for readability. The theory presented here includes the effects of guided and evanescent modes as well. The inclusion of the latter leads to unphysical divergence of the radiated power, if the dipole is lying in a layer with loss or gain. As discussed in this paper, this may be overcome by a so-called regularization procedure [17].

The rest of this paper is organized as follows. In Sec. II, basic equations and the used notation are presented. In Sec. III, field solutions for the 2D case are considered, to ease the discussion in Sec. IV where the field solutions for the 3D case are presented and expressions are given for the power radiated by an oscillating dipole. The paper ends with a number of applications of the presented theory in Sec. V, and conclusions in Sec. VI.

## II. BASIC EQUATIONS AND NOTATION

According to the correspondence principle, results of quantum theory have to be identical to that of classical theory in the limit of many particles. So, it seems reasonable [8] to use as a measure of the LDOM the power radiated by a classical dipole. Then, the equations to be solved are Maxwell's curl equations, given by, assuming nonmagnetic media,

$$\nabla \times \mathbf{E} = -i\omega\mu_0\mathbf{H} \tag{1}$$

and

$$\nabla \times \mathbf{H} = i\omega\varepsilon_0\varepsilon_r\mathbf{E} + i\omega\mathbf{P}, \tag{2}$$

where the polarization due to the radiating classical dipole at $\mathbf{r}_s$, with electric dipole moment $\mathbf{p}$, is given by

$$\mathbf{P} = \mathbf{p}\,\delta(\mathbf{r} - \mathbf{r}_s). \tag{3}$$

In the above, the relative permittivity, $\varepsilon_r(\equiv n^2)$, $n$ being the refractive index, defines the layered structure (see Fig. 1) and may be a complex function of $x$, except (as explained in Sec. III) in the two outermost (semi-infinite) layers, which should have no gain. For the dipole as well as for all fields, a time dependence $\exp(i\omega t)$ is assumed but suppressed. The wave equations to be solved follow from Eqs. (1) and (2) and are given by

$$\nabla \times \nabla \times \mathbf{E} = k_0^2\varepsilon_r\mathbf{E} + \omega^2\mu_0\mathbf{P}, \tag{4}$$

 

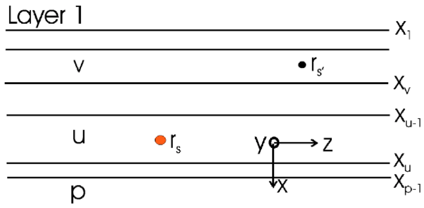


FIG. 1. Considered layer structure consisting of layers 1-$p$, with an oscillating dipole in layer $u$, at $\mathbf{r}_s$. The field is evaluated at $\mathbf{r}_{s'}$, in layer $v$.

$$\varepsilon_r \nabla \times 1/\varepsilon_r \nabla \times \mathbf{H} = k_0^2 \varepsilon_r \mathbf{H} + i\omega \nabla \times \mathbf{P}. \quad (5)$$

The above two equations are equivalent but for simplicity we will choose to solve Eq. (4) for $s$ or TE polarization and Eq. (5) for $p$ or TM polarization.

Once we know the field distribution belonging to a radiating dipole, the outgoing power can be calculated by evaluating the power in the excited modes. However, it is more convenient to consider the power, $P$, escaping from a small sphere around the dipole at $\mathbf{r}_s$ [8]. As discussed below, we have to assume that the dipole-containing layer has a real index in order to get a finite result for the escaping power. The latter is given according to Poynting's theorem by

$$P = \frac{1}{2}\oint_{surface} \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) d\mathbf{s} = \frac{1}{2}\oint_{volume} \mathrm{Re}[\nabla \cdot (\mathbf{E} \times \mathbf{H}^*)] dv, \quad (6)$$

where the asterisk ( $^*$) indicates the complex conjugate and we used the theorem of Gauss for the second equality. Next, using $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$ and Eqs. (1)–(3), it follows that

$$P = -\omega/2\, \mathrm{Im}[\mathbf{E}(\mathbf{r}_s) \cdot \mathbf{p}^*], \quad (7)$$

where we have used that $\varepsilon_r(\mathbf{r}_s)$ is real, so that a contribution to $P$ proportional to $\oint_{volume} \mathrm{Im}(\varepsilon_r |\mathbf{E}|^2) dv$ vanishes.

For later use, we remark that a modal analogue to Eq. (7) can be formulated for guided modes in the case of a structure with real $\varepsilon_r(x)$. Only in that case, due to the orthogonality of modes with the power-related inner product [17], does it follow for the power radiated into the $m$th mode, after a similar derivation as above, that

$$P_m = \frac{1}{2}\oint_{surface} \mathrm{Re}(\mathbf{E}_m \times \mathbf{H}_m^*) d\mathbf{s} = -\omega/2\, \mathrm{Im}[\mathbf{E}_m(\mathbf{r}_s) \cdot \mathbf{p}^*], \quad (8)$$

where the subscript $m$ labels modal field and power.

In order to explain the necessity to choose a real $\varepsilon_r$ in the vicinity of the dipole, we consider first the solutions to Eqs. (4) and (5) for uniform space, with real $\varepsilon_r$. We choose to solve Eq. (5) and use that the operator $-\nabla \times \nabla \times$ can be rewritten into the operator $\Delta (\equiv \partial_{xx} + \partial_{yy} + \partial_{zz})$, as $\nabla \cdot \mathbf{H} = \mathbf{0}$, according to Eq. (1). Assuming only a single component of the polarization, $P_x$, it follows for the Fourier domain

$$(K^2 - k^2)\tilde{H}_{y/z}(\mathbf{k}) = -/+\, \omega k_{z/y} p_x \exp(i\mathbf{k} \cdot \mathbf{r}_s)/(2\pi)^{3/2}. \quad (9)$$

Here $K^2 \equiv k_0^2 \varepsilon_r$, $\mathbf{k} \equiv (k_x, k_y, k_z)^t$, $k \equiv |\mathbf{k}|$, and $\tilde{\mathbf{H}}(\mathbf{k}) \equiv \int \mathbf{H}(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r}) d\mathbf{r}/(2\pi)^{3/2}$. The electric field along $x$ corresponding to Eq. (9) follows with Eq. (2),

$$\tilde{E}_x = -p_x[(k_y^2 + k_z^2)/(K^2 - k^2) + 1] \exp(i\mathbf{k} \cdot \mathbf{r}_s)/[(2\pi)^{3/2} \varepsilon_0 \varepsilon_r]. \quad (10)$$

In order to discuss the radiated power of a dipole in uniform space, we consider the back transform of $\tilde{E}_x$, which can be written according to Eq. (10) as

$$E_x(\mathbf{r}) = A - P_x/(\varepsilon_0 \varepsilon_r),$$

$$A \equiv -\int p_x (k_y^2 + k_z^2)/(K^2 - k^2) \times \exp[i\mathbf{k} \cdot (\mathbf{r}_s - \mathbf{r})] d\mathbf{k}/(8\pi^3 \varepsilon_0 \varepsilon_r). \quad (11)$$

Using that $(k_y^2 + k_z^2)/(K^2 - k^2) + 1 = (K^2 - k_x^2)/(K^2 - k^2)$, it follows also from Eq. (10) that

$$E_x(\mathbf{r}) = -K^2 B p_x/(\varepsilon_0 \varepsilon_r) - A/2,$$

$$B \equiv \int 1/(K^2 - k^2) \exp[i\mathbf{k} \cdot (\mathbf{r}_s - \mathbf{r})] d\mathbf{k}/(8\pi^3), \quad (12)$$

where [19] $B = \exp(-iK|\mathbf{r} - \mathbf{r}_s|)/(4\pi|\mathbf{r} - \mathbf{r}_s|)$, with $K > 0$ as only outgoing waves are considered. Combining Eqs. (11) and (12), it follows that

$$E_x(\mathbf{r}) = [-K^2 \exp(-iK|\mathbf{r} - \mathbf{r}_s|)/(2\pi|\mathbf{r} - \mathbf{r}_s|) - \delta(\mathbf{r} - \mathbf{r}_s)] p_x/(3\varepsilon_0 \varepsilon_r). \quad (13)$$

As can be seen, the solution blows up at $\mathbf{r} = \mathbf{r}_s$, and contains even a term with a $\delta$ function, which would lead to an infinite radiated power if $\varepsilon_r$ would be complex. However, for real $\varepsilon_r$ the radiated power is finite [8] and is, according to Eqs. (7) and (13), given by

$$P = \omega k_0^3 n |p_x|^2/(12\pi\varepsilon_0). \quad (14)$$

If $\varepsilon_r$ were not real, a term proportional to $\oint_{volume} \mathrm{Im}(\varepsilon_r |\mathbf{E}|^2) dv$ would have to be added to Eq. (14). But, as can be seen from Eq. (13), this term would also cause the radiated power to become infinite. The above artifacts occurring for complex $\varepsilon_r$ values are attributed to the (unphysical) choice of point sources and probably also to the use of the classical picture of uniformly absorbing or amplifying layers. A treatment including extended sources would be interesting but is beyond the scope of this paper. A simpler option to avoid the above artifacts is the multiplication of the right-hand side of Eq. (10) with a regularization term [17] given by $\Lambda^4/(\Lambda^4 + k^4)$, where $1/\Lambda$ corresponds to a certain length. As a consequence of that, the effects of rapidly oscillating fields, i.e., evanescent field solutions, are diminished, leading to a finite value for the radiated power.

For real structures, it can be shown (in agreement with the above correspondence principle), as discussed below, that

$$\rho_x/\rho_{x,0} = P(p_x)/P_0(p_x), \tag{15}$$

where $\rho_x$ denotes the LDOM corresponding to the $x$-polarized source, the subscript *0* indicates uniform space, where we will assume the same index as that of the source-containing layer, and we have assumed a source with $x$ polarization. For structures with real refractive indices, the radiated power according to Eq. (7) is a measure for the LDOM, which can be seen as follows. For real structures, the LDOM (or local density of states) is given by [19]

$$\rho_x(\mathbf{r}) = -\,\mathrm{Im}[G_{xx}(\mathbf{r}_s,\mathbf{r}=\mathbf{r}_s,k_0^2,\varepsilon_r)]/\pi. \tag{16}$$

Here $G_{xx}$ is the $x$ component corresponding to an $x$-polarized source term of the Green's tensor, which for the considered problem is defined by [see Eq. (4)]

$$(k_0^2 - 1/\varepsilon_r \nabla \times \nabla \times)\mathbf{G} = \mathbf{I}\,\delta(\mathbf{r}-\mathbf{r}_s), \tag{17}$$

with $\mathbf{I}$ the unit matrix. Comparing Eqs. (7), (16), and (17), it can be seen that (for real refractive indices) $P(p_x)$, as calculated with Eq. (7), is indeed proportional to $\rho_x$. With the above, we are able to find an expression for $\rho_{x,0}$. From Eq. (17), after deriving $G_{xx}$ in a similar way as Eqs. (13) and (16), it follows that

$$\rho_{x,0}(k_0^2,\varepsilon_r) = k_0 n^3/(6\pi). \tag{18}$$

Here $\rho_{x,0}$ is the number of states corresponding to (here) $x$ polarization per unit volume and per unit of $k_0^2$. The LDOM per unit of angular frequency, $\omega$ (a quantity more directly related to the photon energy), is then $\rho_{x,0}(\omega,\varepsilon_r) = n^3\omega^2/(3\pi^2c^3)$.

Next, we make a few remarks about the notation used in the paper. The field solutions to Eqs. (4) and (5) for layered structures in, say, layer $q$ contain, for a given wave vector parallel to the interfaces, $\mathbf{k}_\parallel(\equiv\mathbf{k}_y+\mathbf{k}_z)$ terms of the form $\exp(\pm\gamma_q x)$, where

$$\gamma_q \equiv \gamma_q' + ik_{x,q} \equiv \sqrt{k_\parallel^2 - k_0^2 n_q^2}, \tag{19}$$

where $k_\parallel = |\mathbf{k}_\parallel|$ and $n_q^2(\equiv\varepsilon_{r,q})$ is the squared refractive index of that layer. Throughout this paper, the sign of the square root in Eq. (19) is chosen such that $\gamma_q'$ is positive, unless $\gamma_q' = 0$ (which may occur for real refractive indices), in which case $k_{x,q}$ is chosen positive.

As will be shown below, field solutions to the above equations can be expressed using reflection ($r$) and transmission ($t$) coefficients, which will be denoted as, e.g., $r_{qu}$ and $t_{qu}$, where the subscripts indicate the considered layer stack. For example, $t_{qu}$ is the coefficient for the transmission of an incoming beam in layer $q$ to layer $u$, with a decaying field in layer $u$. To simplify the notation, in this paper we will use the subscript $s$ to indicate reflection and transmission with the $x$-position of the source as reference, e.g., in the case that $u\neq 1$ and $u\neq p$, with layer $u$ containing the source (see Fig. 1),

$$r_{s1} = \exp[-2\gamma_u(x_s - x_{u-1})]r_{u1} \tag{20}$$

and

$$t_{sp} = \exp[-\gamma_u(x_u - x_s)]t_{up}. \tag{21}$$

## III. FIELD SOLUTIONS FOR THE 2D CASE

Below we will derive field solutions corresponding to a radiating dipole in a layered structure, for the 2D case, assuming $\partial_y \equiv 0$. The polarization is given by

$$\mathbf{P}_{2\mathrm{D}} = \mathbf{p}_{2\mathrm{D}}\,\delta(x-x_s)\,\delta(z-z_s). \tag{22}$$

Considering TE (s) polarization first, the only electric field component is along $y$ and we choose to solve Eq. (4), which leads with Eq. (22) to

$$(\partial_{xx} + \partial_{zz} + k_0^2 n^2)E_y = -\,\omega^2\mu_0 P_{2\mathrm{D},y}. \tag{23}$$

The Fourier transform of Eq. (23) with respect to $z$ is given by

$$(\partial_{xx} - k_z^2 + k_0^2 n^2)\tilde{E}(k_z,x) = -\,C_1\,\delta(x-x_s) \tag{24}$$

with $\tilde{E}(k_z,x) \equiv \int_{-\infty}^{\infty} \exp(ik_z z)E_y(x,z)dz/\sqrt{2\pi}$ and $C_1 \equiv \omega^2\mu_0 p_{2\mathrm{D},y}\exp(ik_z z_s)/\sqrt{2\pi}$.

Equation (24) is solved for the field inside layer $u$ along the following lines: (i) the $\delta$ function in Eq. (24) is approximated by $1/\xi$ in the interval $x_s-\xi/2 < x < x_s+\xi/2$; (ii) in that interval the field solution is the sum of the homogeneous and inhomogeneous solutions to Eq. (24) given by

$$a_+\exp[\gamma_u(x-x_s)] + a_-\exp[-\gamma_u(x-x_s)] + b;$$

(iii) the field solutions for $x_{u-1} < x < x_s - \xi/2$ and $x_u > x > x_s + \xi/2$ are proportional to $\exp[\gamma_u(x-x_s)] + r_{s1}\exp[-\gamma_u(x-x_s)]$ and $\exp[-\gamma_u(x-x_s)] + r_{sp}\exp[\gamma_u(x-x_s)]$, respectively; and (iv) next, demanding that at the interfaces $x = x_s \pm \xi/2$ the ratio $\tilde{E}/\partial_x\tilde{E}$ is continuous, it follows after taking the limit $\xi \to 0$ that

$$\tilde{E}(k_z,x) = C_2(1+r_{sp})\{\exp[\gamma_u(x-x_s)] + r_{s1}\exp[-\gamma_u(x-x_s)]\}, \quad x_{u-1} < x < x_s, \tag{25}$$

$$\tilde{E}(k_z,x) = C_2(1+r_{s1})\{\exp[-\gamma_u(x-x_s)] + r_{sp}\exp[\gamma_u(x-x_s)]\}, \quad x_s < x < x_u, \tag{26}$$

where $C_2 \equiv C_1/[2\gamma_u(1-r_{s1}r_{sp})]$. The results allow for a simple physical interpretation. For uniform space ($r_{s1}=r_{sp}=0$), the field solutions correspond to up- and down-going plane waves, both with amplitude $C_1/(2\gamma_u)$. Next, taking into account multiple reflections, the solutions (25) and (26) follow. For example, the amplitude of the up-going field just above $x_s$ follows from $C_1(1+r_{sp})(1+r_{s1}r_{sp}+r_{s1}^2r_{sp}^2\ldots)/(2\gamma_u) = C_2(1+r_{sp})$.

From the above, the field solutions at any $x$ value can be derived with standard reflection and transmission laws, leading to

$$\tilde{E}(k_z, x_{s'}) = fC_1/(2\gamma_u), \tag{27}$$

with

$$f \equiv t_{ss'}N_{s'1}^+N_{sp}^+/(D_{s1,sp}D_{s'1,s'u}), \quad x_{s'} < x_s \tag{28}$$

and

$$f \equiv t_{ss'} N_{s1}^{+} N_{s'p}^{+} / (D_{s1,sp} D_{s'p,s'u}), \quad x_{s'} > x_s. \tag{29}$$

For the above, and also for later, we introduced the following definitions:

$$N_{\sigma q}^{\pm} \equiv 1 \pm r_{\sigma q}, \quad D_{\sigma q,\sigma w} \equiv 1 - r_{\sigma q} r_{\sigma w},$$

$$\sigma = s \text{ or } s'; \quad q,w = 1 \text{ or } p, \tag{30}$$

and, to facilitate the notation, a subscript $s'$ to indicate the considered $x$ value. Note that, if $v=u$, $D_{s'1,s'u}=D_{s'p,s'u}=1$, as then $r_{s'u}=0$, and that then Eq. (27) agrees with Eqs. (25) and (26), as it should. The latter can be shown using equations like ($x_{s'}$ is inside layer $u$) $r_{s'u}=0$ and $t_{ss'}=\exp[-\gamma_u|x_{s'}-x_s|]$.

For TM ($p$) polarization, we choose Eq. (5) as a starting point, as then, for the 2D case with $\partial_y \equiv 0$, only a single magnetic field component, $H_y$, is involved. The equation to be solved, for the considered layer system, is as follows:

$$\hat{O} H_y = -i\omega \{\nabla \times \mathbf{p}_{2D} \delta(x-x_s)\delta(z-z_s)\}_y,$$

$$= i\omega \{\nabla_s \times \mathbf{p}_{2D} \delta(x-x_s)\delta(z-z_s)\}_y, \tag{31}$$

with

$$\hat{O} \equiv \varepsilon_r \partial_x 1/\varepsilon_r \partial_x + \partial_{zz} + k_0^2 \varepsilon_r$$

where the subscript in $\nabla_s$ indicates differentiation with respect to $x_s$ and $z_s$.

Introducing the potential $\mathbf{A}[\equiv(A_x,A_z)]$ with

$$H_y = \{\nabla_s \times \mathbf{A}\}_y, \tag{32}$$

it follows that

$$\hat{O} A_{x/z} = i\omega p_{2D,x/z} \delta(x-x_s)\delta(z-z_s), \tag{33}$$

where we have used that the operator $\nabla_s$ (unlike $\nabla$) commutes with $\hat{O}$. The two equations (33) can be solved, like Eq. (23), by considering again the Fourier transform with respect to $z$. The result is

$$\tilde{A}_{x/z}(k_z,x_{s'}) = f C_{3,x/z}/(2\gamma_u), \tag{34}$$

where $C_{3,x/y} \equiv -i\omega p_{2D,x/z} \exp(ik_z z_s)/\sqrt{2\pi}$ and $f$ is defined by Eqs. (28) and (29), whereby now the reflection and transmission coefficients correspond to TM polarization. The expression for the magnetic field solutions can be found using Eq. (32),

$$\tilde{H}_y(k_z,x_{s'}) = (ik_z f C_{3,x} - \gamma_u C_{3,z} g)/(2\gamma_u), \tag{35}$$

where $g \equiv (\partial f/\partial x_s)/\gamma_u$ is given by

$$g \equiv -t_{ss'} N_{s'1}^{+} N_{sp}^{-} / (D_{s1,sp} D_{s'1,s'u}), \quad x_{s'} < x_s \tag{36}$$

and

$$g \equiv t_{ss'} N_{s1}^{-} N_{s'p}^{+} / (D_{s1,sp} D_{s'p,s'u}), \quad x_{s'} > x_s. \tag{37}$$

To derive Eqs. (36) and (37), we have used equations like Eqs. (20) and (21) and also that the denominator in $f$ does not depend on $x_s$, as long as the source remains in layer $u$.

Next, expressions for the corresponding electric fields can be derived using Eq. (2). The results are, for $x_s \neq x_{s'}$,

$$\tilde{E}_x(k_z,x_{s'}) = k_z \tilde{H}_y(k_z,x_{s'})/(\omega\varepsilon_0\varepsilon_{r,v}) \tag{38}$$

and

$$\tilde{E}_z(k_z,x_{s'}) = \gamma_v (k_z C_{3,x} h_1/\gamma_u + iC_{3,z} h_2)/(2\omega\varepsilon_0\varepsilon_{r,v}), \tag{39}$$

where $h_1 \equiv (\partial f/\partial x_{s'})/\gamma_v$ is given by

$$h_1 = t_{ss'} N_{s'1}^{-} N_{sp}^{+} / (D_{s1,sp} D_{s'1,s'u}), \quad x_{s'} < x_s \tag{40}$$

and

$$h_1 \equiv -t_{ss'} N_{s1}^{+} N_{s'p}^{-} / (D_{s1,sp} D_{s'p,s'u}), \quad x_{s'} > x_s. \tag{41}$$

In Eq. (39), $h_2$ is defined by $\partial g/\partial x_{s'} \equiv \gamma_v h_2 + 2\delta(x_{s'}-x_s)$, with

$$h_2 = -t_{ss'} N_{s'1}^{-} N_{sp}^{-} / (D_{s1,sp} D_{s'1,s'u}), \quad x_{s'} < x_s \tag{42}$$

and

$$h_2 \equiv -t_{ss'} N_{s1}^{-} N_{s'p}^{-} / (D_{s1,sp} D_{s'p,s'u}), \quad x_{s'} > x_s. \tag{43}$$

Next we discuss how to evaluate the back transform of the field solution Eq. (27); those corresponding to Eqs. (38) and (39) can be found similarly. Considering Eq. (27), it follows that

$$E_y(x_{s'},z_{s'}) = C_4 \int_{-\infty}^{\infty} dk_z f \exp[ik_z(z_s - z_{s'})]/\gamma_u,$$

$$C_4 \equiv -\omega^2 \mu_0 p_{2D,y}/(4\pi). \tag{44}$$

Any integration path between $k_z=-\infty$ and $k_z=\infty$ will lead to a solution of Eq. (23), as long as poles and branch cuts are avoided. Below, we will discuss which integration path leads to outgoing field solutions. Poles of $f$, which correspond to modes, occur in the complex $k_z$ plane for zeros of the term $D_{s1,sp}$ ($u \neq 1$ or $p$) occurring in Eqs. (28) and (29). If $u=1$ or $p$ ($D_{s1,sp}=1$), modes correspond to poles of $r_{sp}$ or $r_{s1}$, respectively. The term $D_{s'1,s'u}$ (and equivalently $D_{s'p,s'u}$) does not introduce zeros in the denominator of $f$, as will be made plausible at the end of this section. Branch cuts of $f/\gamma_u$ occur in so-called open systems, with semi-infinite layers 1 and $p$, due to the double-valuedness of $\gamma_{1/p}$ occurring implicitly in $f/\gamma_u$. It can be shown, by considering the expressions for reflection and transmission occurring in $f$, that such branch cuts do not occur for $\gamma_q$, with $q$ indicating one of the inner layers, i.e., $q \neq 1$ or $p$. This is related to the fact that in the field solutions of the inner layers, both solutions $\exp(\pm i\gamma_q x)$ occur. The latter holds also for the outermost layers if a closed system (e.g., with electric walls as boundaries) is considered.

We first consider a uniform (2D) open structure, with a real index $n$, and for simplicity it is assumed in the rest of this section that $x_s=z_s=0$. So, we may rewrite Eq. (44) as

$$E_y(x_{s'},z_{s'}) = C_4 \int_{-\infty}^{\infty} dk_z \exp(-ik_z z_{s'} - \gamma|x_{s'}|)/\gamma. \tag{45}$$

As we are looking for outgoing field solutions, which should decay for large $|x_{s'}|$, it seems natural to choose the integration path denoted by $C_{out}$ in Fig. 2 and $\gamma \in \mathrm{I}$, where we use

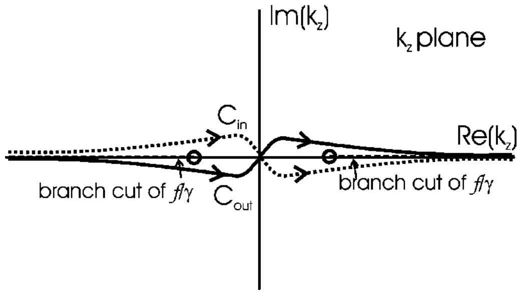


FIG. 2. Integration path $C_{out}$ for the evaluation of outgoing fields in a uniform structure. The path $C_{in}$ can be used to find the incoming fields (see text).

"I–IV" to indicate the four quadrants. The result is (see [20] for a related formula)

$$E_y(x_{s'},z_{s'}) = \pi C_4 H_0^{(2)}(k_0 n\rho), \quad \rho \equiv \sqrt{x_{s'}^2 + z_{s'}^2}, \tag{46}$$

i.e., the field is proportional to the zero-order Hankel function of the second kind, which is indeed an outgoing field solution for the assumed time dependence $\exp(i\omega t)$. Similarly, integration along path $C_{in}$ in Fig. 2, with $\gamma \in$ IV, leads to incoming field solutions, proportional to the zero-order Hankel function of the first kind.

For layered structures, poles in $f$ may occur on, below, or above the real $k_z$ axis if loss and gain are considered. Note that these poles always occur in pairs, $k_z = \pm\beta_m$ (with $\beta_m$ a propagation constant and $m$ labels the mode), due to the nature of the modal field equation, corresponding to Eq. (23) [18]. It is of importance to choose the correct integration path relative to these poles in order to get the desired (outgoing) field solution. Below, we will make plausible that the path should lie above the poles in I and below the poles in III. To get rid of the branch cuts related to $\gamma_{1/p}$, a system closed with electric walls ($r_w = -1$), at a large distance, is considered. As a consequence, the continuous spectrum of the radiation modes becomes discrete and poles appear (in $f$), approximately at positions indicated in Fig. 3 (see also [21,22]). The effect of the introduction of the walls on $f$ is that the reflection coefficients $r_{21}$ and $r_{p-1,p}$ (occurring implicitly in $f$) are altered, e.g., $r_{21}$ has to be replaced by

$$\tilde{r}_{21} = [r_{21} + r_w \exp(-2\gamma_1 d_1)]/[1 + r_{21} r_w \exp(-2\gamma_1 d_1)], \tag{47}$$

with $d_1$ the thickness of layer 1. Note that $\tilde{r}_{21}$ is indeed single-valued, i.e., it remains unchanged if $\gamma_1$ [also in $r_{21} = (\gamma_2 - \gamma_1)/(\gamma_2 + \gamma_1)$ for TE, e.g.] changes sign, and also that $\tilde{r}_{21} \to r_{21}$ if $d_1 \to \infty$, if $\gamma_1 \in$ I. The latter choice can always be made if $\varepsilon_1 \in$ III or IV, including the real axis, which implies that (neglecting negative refractive indices) $n_1$ has to be positive real or in IV (i.e., absorbing). As a consequence of the above, the integrals along $C_{out}$ (see Fig. 3) are equivalent for the open and closed systems, if $d_1 \to \infty$.

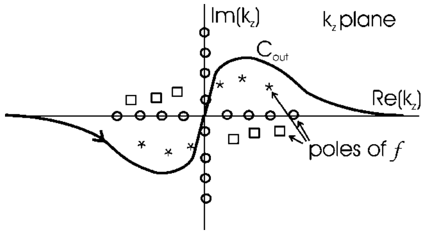


FIG. 3. Typical locations of the poles of $f$ for a real structure (○), an absorbing structure (squares), and a structure with gain ( $^*$). Fields can be evaluated by integrating along path $C_{out}$, or along the part lying in the first quadrant, $C_{out}^I$ (see text).

If $z_{s'} > 0$ ($<0$), the path $C_{out}$ can be closed with a semicircle with infinite radius in the lower (upper) complex $k_z$ plane, thus encircling the poles in I and IV (II and III). Then, for $x_{s'} \neq 0$ it follows that the integral over the semicircle vanishes [due to the presence of terms $t_{ss'}$ and $\exp(-ik_z z_{s'})$ in the integrand], and so the integral along $C_{out}$ contains the contributions of all poles in I and IV (II and III), which are proportional to $e_m(x_{s'})\exp(-i\beta_m|z_{s'}|)$, with $\beta_m \in$ I or IV or on the negative imaginary axis (for part of the poles if the structure is real) and $e_m(x)$ the corresponding modal field. So, the thus found field is, as desired, a sum of outgoing running modes and (for real structures) decaying evanescent modes. Here we also used that each pole occurring in the integrand of Eq. (44) contributes to the integral with a term proportional to $e_m(x_{s'})$, which can be seen as follows. Considering first layer $u$, it can be shown from Eqs. (28) and (29) that at a pole, where $r_{s1} = 1/r_{sp}$, the residue of $f$ is proportional to $\exp[\gamma_u(x_{s'} - x_{u-1})] + r_{u1}\exp[-\gamma_u(x_{s'} - x_{u-1})] \propto e_m(x_{s'})$, $x_{s'} \in$ layer $u$. From standard reflection and transmission laws, it follows that the same holds for $x_{s'}$ in any other layer, also for the special case that $x_s$ is positioned in one of the outer layers. In a similar way, it can be shown from Eqs. (35)–(37) that at a pole the residues of both $f$ and $g$, occurring in Eq. (35), are proportional to $h_m(x_{s'})$, the corresponding modal field.

From the above, it follows that the field $E_y(x_{s'},z_{s'})$ in layer $u$ is a sum of modal field solutions. The fields in the other layers are continuations of these. These can also be constructed by the transfer matrix model [22], leading to fields that are finite for all $x_{s'}$, which follows from the nature of the matrices involved. As a consequence, the only poles of $f$ [occurring in Eq. (27)] are due to the term $D_{s1,sp}$, and the terms $D_{s'1,s'u}$ or $D_{s'p,s'u}$ do not introduce extra poles.

## IV. FIELD SOLUTIONS AND RADIATED POWER FOR THE 3D CASE

We first consider TE polarization and consider the Fourier transform, with respect to both $y$ and $z$, of the electric field for a certain value of the in-plane wave vector $\mathbf{k}_\parallel \equiv \mathbf{k}_y + \mathbf{k}_z$. The coordinate system is rotated around the $x$ axis such that the $z'$ axis has the same direction as $\mathbf{k}_\parallel$ (see Fig. 4). Then, for TE polarization the electric field corresponding to $\mathbf{k}_\parallel$ is directed along $y'$ and it follows according to a derivation as for Eq. (27) that

$$\tilde{E}_{y'}^{\mathrm{TE}}(\mathbf{k}_\parallel, x_{s'}) = f(k_\parallel) C_5 \exp(i\mathbf{k}_\parallel \cdot \boldsymbol{\rho}_s)/(2\gamma_u), \tag{48}$$

with $C_5(k_\parallel) = \omega^2\mu_0(p_y\cos\theta + p_z\sin\theta)/(2\pi)$, $k_\parallel \equiv |\mathbf{k}_\parallel|$, and $\boldsymbol{\rho}_s \equiv (0 y_s z_s)^t$. The electric field at a position $\mathbf{r}_{s'}$ can now be derived by back transformation of Eq. (48). The result is

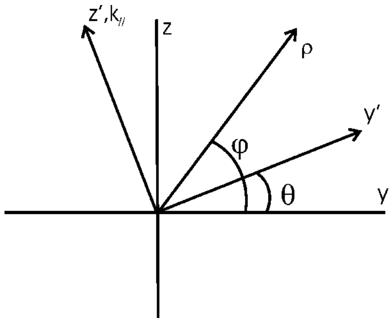


FIG. 4. Definitions used for the rotation of the coordinate system along $x$, and for the transformation to a cylindrical coordinate system.

$$E^{\mathrm{TE}}_{y/z}(\mathbf{r}_{s'}) = \left\{ \int_{-\infty}^{\infty} dk_y dk_z \exp(-i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho}) f C_{6,y/z}/\gamma_u \right\} \Big/ (4\pi), \tag{49}$$

where $C_{6,y} \equiv C_5 \cos\theta$ and $C_{6,z} \equiv C_5 \sin\theta$, and $\boldsymbol{\rho} \equiv \boldsymbol{\rho}_{s'} - \boldsymbol{\rho}_s$. Changing to a cylindrical coordinate system for $k_y$ and $k_z$ leads to

$$E^{\mathrm{TE}}_{y/z}(\mathbf{r}_{s'}) = \int_0^{\infty} dk_{\parallel} k_{\parallel} f/\gamma_u \int_0^{2\pi} d\theta C_{6,y/z} \times \exp[i k_{\parallel}\rho \sin(\theta-\varphi)]/(4\pi), \tag{50}$$

where $\varphi$ is the angle between $\boldsymbol{\rho}$ and the positive $y$ axis (see Fig. 4). The integration over $\theta$ can be performed analytically. Writing the dipole moment also in cylindrical coordinates, and using the relation for the zero-order Bessel function $J_0(\xi) = \int_0^{2\pi} d\alpha \exp(i\xi \sin\alpha)/(2\pi)$ [20], and its derivatives with respect to $\xi$ (to deal with cosine and sine terms in the integrand) and recurrence relations for the Bessel function [20], it follows after some standard manipulations that

$$E^{\mathrm{TE}}_{\rho/\varphi} = \omega^2 \mu_0 p_{\rho/\varphi} \int_0^{\infty} dk_{\parallel} k_{\parallel} f C^{\mathrm{TE}}_{\rho/\varphi}/(4\pi\gamma_u), \tag{51}$$

with $C^{\mathrm{TE}}_{\rho} \equiv J_1(k_{\parallel}\rho)/(k_{\parallel}\rho)$ and $C^{\mathrm{TE}}_{\varphi} \equiv J_0(k_{\parallel}\rho) - J_1(k_{\parallel}\rho)/(k_{\parallel}\rho)$. As explained in [20], the integral (51) can be rewritten in terms of an integral over Hankel functions (of the same order as the substituted Bessel function) over the interval $(-\infty,\infty)$, using $J_q(\alpha) = [H^{(1)}_q(\alpha) + H^{(2)}_q(\alpha)]/2$ and expressions relating $H^{(1)}_q(\alpha)$ and $H^{(2)}_q(-\alpha)$, $q = 0,1,\ldots$ [23]. For example, for the field along $\rho$ it follows that

$$E^{\mathrm{TE}}_{\rho} = \omega^2 \mu_0 p_{\rho} \int_{-\infty}^{\infty} dk_{\parallel} f H^{(2)}_1(k_{\parallel}\rho)/(8\pi\rho\gamma_u), \tag{52}$$

where the origin of the $k_{\parallel}$ plane should be avoided [20] as there the Hankel function has a singularity. In light of the discussion at the end of Sec. III, it can be seen that the integration path for Eq. (52) has to be $C_{out}$, as defined above (but avoiding the origin). Then, with similar arguments as above, and considering the asymptotic behavior of $H^{(2)}_q$ for large $\rho$, it follows that for a closed system the field (52) consists of outgoing modes. The fields of the separate modes can be obtained by encircling the corresponding poles on integrating. The total field can be evaluated from Eq. (51) with the integration path the part of $C_{out}$ lying in $I$, say $C^{I}_{out}$. The above holds also for the open system, which may be considered, with respect to the considered integrals, as a limiting case $(d_{1/p}\to\infty)$ of the closed system.

Using Eqs. (51), (7), and $J_0(0)=1$, $J_1(\xi)/\xi|_{\xi=0} = 1/2$, the total radiated power for TE polarization can be expressed as

$$P^{\mathrm{TE}} = -\omega^3 \mu_0/(16\pi)\, \mathrm{Im}\left( \int_0^{\infty} dk_{\parallel} k_{\parallel} f_s |p_{\parallel}|^2/\gamma_u \right), \tag{53}$$

where $f_s \equiv N^{+}_{s1} N^{+}_{sp}/D_{s1,sp}$ corresponds to $f(x_s = x_{s'})$ according to Eq. (28), and $|p^2_{\parallel}| \equiv |p^2_y| + |p^2_z|$.

In order to find an expression for the power going into TM modes, we choose to solve Eq. (5), which can be written for the considered planar structure as

$$\hat{O}\mathbf{H} = -i\omega \boldsymbol{\nabla} \times \mathbf{P}, \quad \hat{O} \equiv \varepsilon_r \partial_x 1/\varepsilon_r \partial_x + \partial_{yy} + \partial_{zz} + k_0^2 \varepsilon_r, \tag{54}$$

and in the Fourier domain as

$$\hat{O}\tilde{H}_{y'}(\mathbf{k}_{\parallel}, x_{s'}) = i\omega\{\boldsymbol{\nabla}_s \times \mathbf{p}\,\delta(x_{s'} - x_s)\exp(i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho}_s)\}_{y'}/(2\pi), \tag{55}$$

with $\boldsymbol{\nabla}_s$ as defined below Eq. (33). As in Sec. III, we define a potential $\tilde{\mathbf{A}}[\equiv(\tilde{A}_x 0 \tilde{A}_{z'})^t]$, such that $\tilde{H}_{y'} = \{\boldsymbol{\nabla}_s \times \tilde{\mathbf{A}}\}_{y'}$, so that we may write

$$\hat{O}\tilde{\mathbf{A}}(\mathbf{k}_{\parallel}, x_{s'}) = C_7 \mathbf{p}\,\delta(x_{s'} - x_s), \quad C_7 = i\omega \exp(i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho}_s)/(2\pi). \tag{56}$$

Equation (56) can be solved for both the $x$ and $z'$ vector components, leading to

$$\tilde{\mathbf{A}}(\mathbf{k}_{\parallel}, x_{s'}) = -C_7 \mathbf{p} f/(2\gamma_u). \tag{57}$$

The corresponding magnetic field solution is then

$$\tilde{H}_{y'}(\mathbf{k}_{\parallel}, x_{s'}) = C_7(-ik_{\parallel} p_x f + \gamma_u p_{z'} g)/(2\gamma_u), \tag{58}$$

where $p_{z'} = p_z \cos\theta - p_y \sin\theta$. The corresponding electrical fields can be calculated from Eq. (2), leading to, for $x_s \neq x_{s'}$,

$$\tilde{E}^{\mathrm{TM}}_{x}(\mathbf{k}_{\parallel}, x_{s'}) = k_{\parallel}\tilde{H}_{y'}/(\omega\varepsilon_0\varepsilon_{r,v}) \tag{59}$$

and

$$\tilde{E}^{\mathrm{TM}}_{z'}(\mathbf{k}_{\parallel}, x_{s'}) = C_8(-ik_{\parallel} h_1 p_x/\gamma_u + h_2 p_{z'})/(\varepsilon_0\varepsilon_{r,v}), \tag{60}$$

where $C_8 \equiv \gamma_v \exp(i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho})/(4\pi)$. After back transforming the above, it follows, for $x_s \neq x_{s'}$, that

$$E^{\mathrm{TM}}_{x}(\mathbf{r}_{s'}) = \int_{-\infty}^{\infty} dk_y dk_z (k^2_{\parallel} f p_x/\gamma_u - ik_{\parallel} g p_{z'}) \times \exp(-i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho})/(8\pi^2)/(\varepsilon_0\varepsilon_{r,v}), \tag{61}$$

$$E_{y/z}^{\rm TM}(\mathbf{r}_{s'}) = \int_{-\infty}^{\infty} dk_y dk_z C_{y/z}\gamma_v(ik_\parallel h_1 p_x/\gamma_u - h_2 p_{z'}) \times \exp(-i\mathbf{k}_\parallel\cdot\boldsymbol{\rho})/(8\pi^2)/(\varepsilon_0\varepsilon_{r,v}) \quad (62)$$

with $C_y=\sin\theta$ and $C_z\equiv-\cos\theta$. Similar to the TE case, Eqs. (61) and (62) can be written in cylindrical coordinates, which leads to ($\mathbf{r}_s\neq\mathbf{r}_{s'}$)

$$E_x^{\rm TM}(\mathbf{r}_{s'}) = \int_0^{\infty} dk_\parallel(k_\parallel^3 f p_x J_0/\gamma_u + k_\parallel^2 g p_\rho J_1)/(4\pi\varepsilon_0\varepsilon_{r,v}), \quad (63)$$

$$E_\rho^{\rm TM}(\mathbf{r}_{s'}) = \int_0^{\infty} dk_\parallel\gamma_v\{k_\parallel h_2 p_\rho[J_0 - J_1/(k_\parallel\rho)] - k_\parallel^2 h_1 p_x J_1/\gamma_u\}/(4\pi\varepsilon_0\varepsilon_{r,v}), \quad (64)$$

$$E_\varphi^{\rm TM}(\mathbf{r}_{s'}) = \int_0^{\infty} dk_\parallel\gamma_v k_\parallel h_2 p_\varphi J_1/(k_\parallel\rho)/(4\pi\varepsilon_0\varepsilon_{r,v}). \quad (65)$$

Until now we did not bother about the contribution of terms with $\delta$ functions to the field solutions at $\mathbf{r}_s=\mathbf{r}_{s'}$. Instead of going through the calculations again, we consider the field solutions (51) and (63)–(65) for uniform space (i.e., all reflection coefficients are zero) and compare the result with field solution (13). In so doing, it appears that we have to add the terms $-P_x/(\varepsilon_0\varepsilon_{r,u})$ and $-P_{y/z}/(3\varepsilon_0\varepsilon_{r,u})$ to the thus far given field solutions, with the corresponding polarization.

From Eqs. (63)–(65) and (7), we derive for the outgoing power for TM polarization

$$P^{\rm TM} = -\omega\,{\rm Im}\left[\int_0^{\infty} dk_\parallel(k_\parallel^3 f_s|p_x|^2/\gamma_u + k_\parallel\gamma_u h_{2,s}|p_\parallel|^2/2)\right]\Big/(8\pi\varepsilon_0\varepsilon_r), \quad (66)$$

where $h_{2,s}\equiv-N_{s1}^{-}N_{sp}^{-}/D_{s1,sp}$. Equations equivalent to Eqs. (53) and (66) have been published before by [8,11] for the evaluation of the power radiated into radiation modes and by [16] for SE rates in lossy layered structures.

As a final remark, we note that the principle of reciprocity applies [20] to the field solutions given above. That is, the field component at $\mathbf{r}_{s'}$ along $\xi(\xi=x,\rho,\varphi)$ due to a dipole at $p_\zeta(\mathbf{r}_s)$, $(\zeta=x,\rho,\varphi)$ is equal to the field along $\zeta$ at $\mathbf{r}_s$ due to a dipole $p_\xi(\mathbf{r}_{s'})$ with $p_\xi(\mathbf{r}_{s'})=p_\zeta(\mathbf{r}_s)$. As an example, we will show this for the TM case with $\xi=\rho$ and $\zeta=x$ with $x_{s'}<x_s$. Then, we have to compare the last term of Eq. (63) with that of Eq. (64). In the latter, we have to interchange $\mathbf{r}_s$ and $\mathbf{r}_{s'}$, as now the source is at $\mathbf{r}_{s'}$, and if we want to maintain the definition $\boldsymbol{\rho}\equiv\boldsymbol{\rho}_{s'}-\boldsymbol{\rho}_s$ it follows that we have to introduce a minus sign in $E_\rho^{\rm TM}$. So, in order to prove reciprocity for the considered example, it is sufficient to show that

$$\gamma_v h_1(\mathbf{r}_{s'},\mathbf{r}_s)/(\gamma_u\varepsilon_{r,v}) = g(\mathbf{r}_s,\mathbf{r}_{s'})/\varepsilon_{r,u}, \quad (67)$$

where the second arguments in $h_1$ and $g$ indicate the position of the dipole. From the definitions (37) and (40), we can rewrite Eq. (67) as follows:

$$\gamma_v t_{ss'}N_{s'1}^{-}N_{sp}^{+}/(\gamma_u\varepsilon_{r,v}D_{s1,sp}D_{s'1,s'u}) = t_{s's}N_{s'1}^{-}N_{sp}^{+}/(\varepsilon_{r,u}D_{s'1,s'p}D_{sp,sv}). \quad (68)$$

We first note that the following holds:

$$\varepsilon_{r,u}^p\gamma_v t_{ss'} = \varepsilon_{r,v}^p\gamma_u t_{s's}, \quad p=0\ ({\rm TE})\ {\rm or}\ 1\ ({\rm TM}), \quad (69)$$

a property that can be proved by induction starting with a two-layer stack. Other required properties follow from standard theory,

$$r_{s1} = r_{sv} + t_{ss'}t_{s's}r_{s'1}/(1 - r_{s'1}r_{s'u}), \quad (70)$$

$$r_{s'p} = r_{s'u} + t_{ss'}t_{s's}r_{sp}/(1 - r_{sp}r_{sv}). \quad (71)$$

Substituting Eqs. (69)–(71) into Eq. (68) shows that Eq. (68) holds.

## V. EXAMPLES AND DISCUSSION

As an illustration of the above formulas and also to discuss the effects of gain and loss, a number of applications of the presented theory will be given. As a first example, a system closed with magnetic walls at $x=\pm d$, with real index $n_1$ in between, is considered. A dipole oscillating along $x$ is positioned at $x=0$, and the wavelength is given by $\lambda=1\ \mu$m (see the inset of Fig. 5). The relative LDOM can be calculated from Eqs. (66), (14), and (15). With $r_{s1}=r_{sp}=r_w\exp(-2\gamma d)$, where $\gamma\equiv\sqrt{k_\parallel^2-k_0^2n_1^2}$ and $r_w=-1$ is the reflection coefficient of the (perfectly) reflecting magnetic walls, it follows that

$$\rho_x/\rho_{x,0} = P^{\rm TM}(p_x)/P_0 = -1.5/(k_0^3n_1^3){\rm Im}\left\{\int_0^{\infty} dk_\parallel f_s k_\parallel^3/\gamma\right\}, \quad (72)$$

where $f_s\equiv2\sinh^2(\gamma d)/\sinh(2\gamma d)$ and the integration runs over the path $C_{out}^I$ defined above. Note that poles of $f_s$ will lie on the real (corresponding to guided modes, if any) and imaginary (corresponding to evanescent modes) $k_\parallel$ axes, so that the contour $C_{out}^I$ may lie slightly above the real $k_\parallel$ axis approaching that axis somewhat right of $k_\parallel=k_0n_1$, where the integrand of Eq. (72) becomes purely real and so no longer contributes to Eq. (72). This way the (removable) singularity at $k_\parallel=k_0n_1$, where $\gamma=0$, is avoided.

An expression alternative to Eq. (72) can be obtained by rewriting the $p_x$-induced part of Eq. (63) as

$$E_x^{\rm TM}(\mathbf{r}_{s'}) = \int_{-\infty}^{\infty} dk_\parallel k_\parallel^3 f p_x H_0^{(2)}(k_\parallel\rho_{s's})/(8\pi\gamma\varepsilon_0\varepsilon_r), \quad (73)$$

using similar arguments as for deriving Eq. (52). Due to the presence of the term $t_{ss'}$ and the asymptotic behavior $H_0^{(2)}\sim\sqrt{2/(\pi k_\parallel\rho_{ss'})}\exp[-i(k_\parallel\rho_{ss'}-\pi/4)]$ for large $k_\parallel\rho_{ss'}$, the integration path ($C_{out}$) of Eq. (73) can be closed by a semicircle with infinite radius in the lower $k_\parallel$ complex plane (see also Sec. III), leading to an integration contour, say, $\tilde{C}_{out}$. The radiated power can be written, according to Eq. (7), as

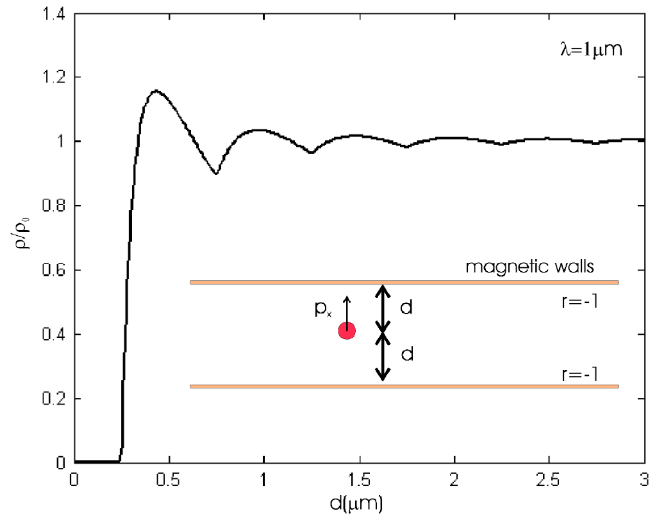


FIG. 5. Relative LDOM for a dipole oscillating along $x$, in the middle of a uniform structure closed with magnetic walls, as a function $d$, with $2d$ the distance between these walls.

$$P^{\mathrm{TM}} = -\omega\, \mathrm{Im}\left[\int_{\tilde{C}_{out}} dk_{\parallel} k_{\parallel}^3 f_s H_0^{(2)}(k_{\parallel}\rho_{s's} \to 0) \times |p_x|^2/\gamma\right] \Big/ (16\pi\varepsilon_0\varepsilon_r). \quad (74)$$

The integral can be rewritten using Gauchy's residue theorem. The poles of $f_s$ occur for $2\gamma d = im\pi$, $m=1,2,\ldots$, and so for $k_{\parallel}$ on the real and imaginary axes. At these poles, it follows for the numerator that $2\{i\sin(m\pi/2)\}^2=-2$ if $m$ is odd and 0 otherwise, corresponding to symmetric and antisymmetric modes, respectively. At the poles corresponding to $m$ odd, the derivative with respect to $k_{\parallel}$ of the denominator of $f_s$ is given by $\partial \sinh(2\gamma d)/\partial k_{\parallel} = [\partial \sinh(2\gamma d)/\partial\gamma]/(\partial\gamma/\partial k_{\parallel}) = -2dk_{\parallel}/\gamma$. As $\mathrm{Re}[H_0^{(2)}(k_{\parallel}\rho_{s's}\to 0)]=1$ for $k_{\parallel}$ on the positive real axis, and 0 for $k_{\parallel}$ on the negative imaginary axis, it follows from Eq. (74) that

$$P^{\mathrm{TM}} = \omega \sum_{\text{real poles}, m \text{ odd}} k_{\parallel}^2/(8d\varepsilon_0\varepsilon_r), \quad (75)$$

where only poles on the positive real $k_{\parallel}$ axis contribute to the radiated power.

The result of applying Eq. (72), or equivalently Eq. (75) with Eq. (7), is given in Fig. 5. At low values of $d$ ($\lesssim 0.2\ \mu$m), there exist no guided modes, and so the radiated power or LDOM is zero. The sudden rise in $\rho_x/\rho_{x,0}$ at $d \approx 0.2\ \mu$m is related to the occurrence of the first guided mode; at higher $d$ values the oscillations in $\rho_x/\rho_{x,0}$ are due to the appearance of higher-order modes. Note that at high $d$ values, $\rho_x/\rho_{x,0}$ approaches unity, i.e., the LDOM of the closed system approaches that of the open system, as may be expected.

As a second example, there is a source in a vacuum layer, sandwiched between two grated structures, corresponding to, if semi-infinite, omnidirectional mirrors [24,25] (see Fig. 6). The considered wavelength, λ=1 $\mu$m, is approximately in the middle of the transmission gap. It is assumed that the dipole oscillates in the $y$-$z$ plane, i.e., both TE and TM waves are excited. As the outermost layers have the highest index ($n_1=n_p=3.4$), the system does not support any guided modes, and all power is eventually radiated into the outermost layers. Figure 7 displays the relative power as a function of the position of the dipole inside the central layer, in the presence of (both twice) a single unit cell ($P=1$) and three unit cells ($P=3$), as calculated with Eqs. (53) and (66), using an integration path in the first quadrant approaching the real $k_{\parallel}$ axis right of $1k_0$ (to avoid the removable singularity, due to $\gamma_u$, at that position) and ending at $3.4k_0$ (where the integrand becomes real, for real $k_{\parallel}$). For large $x_0$ values, oscillations are observed which can be attributed to the presence of quasiguided modes with a considerable field inside the central layer, both for $P=1$ and 3. The difference between these two situations is marginal, indicating that in both cases the reflection coefficient is nearly the same. The increase of the relative radiated power at small $x_0$ is attributed to tunneling to states corresponding to modal indices in the range 1–3.4. This is confirmed by results of calculations using an integration path in the first quadrant of the $k_{\parallel}$ plane, between 0 and $1.01k_0$, as shown in Fig. 7.

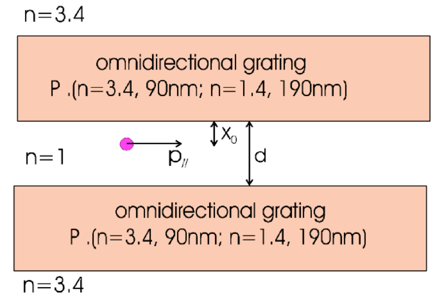


FIG. 6. Structure consisting of a source containing layer, with index $n=1$ and thickness $d=1\ \mu$m, in between two grated structures with both $P$ periods. The wavelength is λ=1 $\mu$m.

As a final example, a three-layer system is considered (see the inset of Fig. 8) with a central guiding layer for different values of the imaginary part of the index, $\mathrm{Im}(n_2)\equiv K = 0, \pm 0.05$. A dipole oscillating along $x$ is assumed in layer 1, at a distance $d$ from the first interface. Both the wavelength and the thickness of the central layer are assumed to be 1 $\mu$m. The total radiated power relative to that of uniform space (with index 1.5) is calculated using Eq. (66) (with a path above the pole belonging to the guided mode) and Eq. (7). The results are given in Fig. 8, as well as the relative power going into the guided mode in the case of real indices. The latter has been calculated in a similar way as Eq. (75), i.e., by rewriting Eq. (63) in terms of $H_0^{(2)}$ and using Eq. (7) and Gauchy's theorem. Note, as discussed in Sec. II, that for structures with loss or gain, modal power is not well defined, as modes are not orthogonal (with a power-related inner product). Considering the relative total radiated power, it can be seen from Fig. 8 that for all three considered cases it goes to unity for larger $d$ values. However, for small $d$ the relative total power becomes larger and larger (smaller and smaller) and will go to (minus) infinity for $K=-0.05$ (0.05). This behavior can be understood from Eq. (66) by noting that for real $k_{\parallel}$,

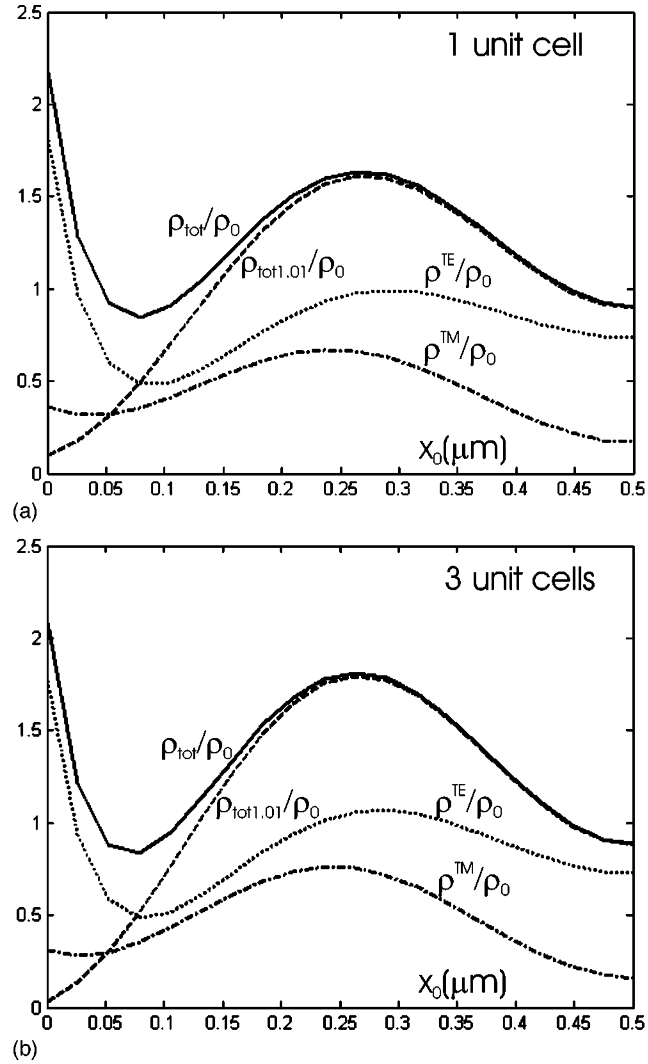


FIG. 7. Relative LDOM as a function of dipole position of the structure given by Fig. 6 with $P=1$ (top figure) and 3 (bottom figure). The subscript 1.01 refers to integration in the first quadrant of the $k_{\|}$ plane between 0 and $1.01k_0$ (see text).

$$\mathrm{Im}(f_s) = \mathrm{Im}(r_{s3}) = \exp(-2\gamma_1 d)\mathrm{Im}(r_{13}) \approx \exp(-2\gamma_1 d)\mathrm{Im}(r_{12}), \tag{76}$$

where the approximation ($r_{13}\approx r_{12}$) holds for large $k_{\|}$, and where we have used that $r_{s1}=0$, as the source lies in layer 1. Using, as follows from the expression for $r_{12}$, that for large $k_{\|}$ $\mathrm{Im}(r_{12})\approx 0.64$ K, it follows indeed that the integral (66) blows up for nonzero $K$ if $d\rightarrow 0$. As the main contributions to the integral come from the region with large $k_{\|}$, the effect is attributed to interactions of the oscillating dipole with evanescent modes. So, for an oscillating dipole, too close to a layer with loss or gain, the relative radiated power is not equal to the relative LDOM, which is assigned to the use of a localized ($\delta$-function-like) source. For an extended source, the overlap of the function describing the source distribution with evanescent modes would rapidly decrease, due to an increase of spatial frequency of the corresponding fields, for increasing $k_{\|}$. A more or less equivalent way to do so is by

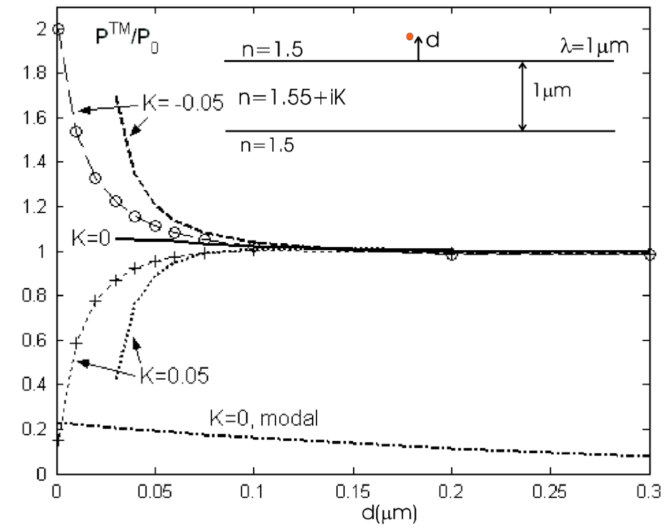


FIG. 8. Relative power radiated by a dipole oscillating along $x$, as a function of its distance, $d$, from the first interface of the considered structure (see inset), for real indices ($K=0$), absorption, and gain ($K=\pm 0.05$). The relative power going into the guided (TM) mode can only be calculated for the case of real indices. The curves with the + and ○ symbols have been evaluated using the regularization procedure (see text) with $\Lambda=25\ \mu\mathrm{m}^{-1}$ for $K=\pm 0.05$.

the regularization procedure [17] mentioned above. By multiplying the integrand of Eq. (66) with $\Lambda^4/(\Lambda^4+k_{\|}^4)$, using $\Lambda=25\ \mu\mathrm{m}^{-1}$, it can be seen in Fig. 8 that for the cases $K=\pm 0.05$, the radiated power no longer diverges if $d\rightarrow 0$.

## VI. CONCLUSIONS

A theory is presented for the evaluation of optical field and power radiated by an oscillating dipole in a layered structure, which may show loss or gain, except the outermost layers of an open structure, which may not show gain. The found expressions are easy to evaluate, without the need of mode-searching routines. By encircling poles corresponding to guided modes on integrating, also modal fields and, for real indices only, modal power can be evaluated. For structures with real indices, the radiated power is a measure for the local density of modes. This does not hold for a source in a layer with loss or gain, where the radiated power becomes infinite. The latter is attributed to interaction of the dipole source with evanescent modes, and may probably be avoided by assuming an extended, rather than a localized, source.

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